## Problem A. Missing Runners

| Input file: | standard input |
| :--- | :--- |
| Output file: | standard output |
| Time limit: | 1 second |
| Memory limit: | 256 megabytes |

You are organizing a marathon with $N$ runners. Every runner is given a distinct number from 1 to $N$, so they can be easily identified.
You record the number of each runner as they cross the finish line. Unfortunately, you notice that only $N-1$ runners have finished. Which runner is still out there?

## Input

The first line of input contains the integer $N\left(1 \leq N \leq 2^{15}\right)$. The next line contains $N-1$ distinct integers in the range from 1 to $N$, representing the numbers of runners who have crossed the finish line.

## Output

Output the number of the runner who has not crossed the finish line.

## Example

|  | standard input |  |  |  | standard output |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 5 |  |  |  | 4 |  |
| 1 | 5 | 3 |  |  |  |

## Problem B. Oblongs and Right Triangles

Input file:
Output file:
Time limit:
Memory limit:
standard input
standard output
1 second
256 megabytes

There are $N$ points on the plane; the $i$-th point is at $\left(X_{i}, Y_{i}\right)$. There may be multiple points at the same location. Four of the points will be coloured black and three other points will be coloured white. The remaining $N-7$ points will be uncoloured.
An oblong is a rectangle that is not a square.
An right triangle is a triangle where one of the interior angles is exactly ninety degrees.
Determine the number of ways to colour the points such that the four black points are the vertices of an oblong and the three white points are the vertices of a right triangle. Note that both shapes should have positive area.

## Input

Line 1 contains one integer $N\left(7 \leq N \leq 2^{4}\right)$.
Line 2 contains $N$ integers $X_{1}, \ldots, X_{N}\left(-2^{29} \leq X_{i} \leq 2^{29}\right)$.
Line 3 contains $N$ integers $Y_{1}, \ldots, Y_{N}\left(-2^{29} \leq Y_{i} \leq 2^{29}\right)$.

## Output

Print one line with one integer, the number of ways to choose an oblong and a right triangle.

## Example

| standard input |  |  |  |  |  |  |  | standard output |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  |  |  |  |  |  |  |  |  |
| -1 | 0 | 0 | 1 | 1 | 1 | 1 | 2 |  |  |
| 0 | 1 | -1 | 0 | -1 | -1 | -2 | -1 |  |  |

## Note

The only way to form an oblong is with points $1,2,7,8$. Of the remaining four points there are two ways to form a right triangle from them.


## Problem C. Unique Substrings

| Input file: | standard input |
| :--- | :--- |
| Output file: | standard output |
| Time limit: | 1 second |
| Memory limit: | 256 megabytes |

Create a string of $N$ lowercase letters $S_{1} S_{2} \ldots S_{N}$ where $1 \leq N \leq 2^{12}$. The string should have exactly $K$ unique substrings.
A substring is the sequence of letters of the form $S_{L} S_{L+1} \ldots S_{R-1} S_{R}$ for some $1 \leq L \leq R \leq N$. Two substrings are the same if they are the same sequence of letters.

## Input

Line 1 contains one integer $K\left(1 \leq K \leq 2^{22}\right)$. $N$ is not given; the string that you create may have any number of letters $N$ as long as $1 \leq N \leq 2^{12}$.

## Output

Print one line with one string of $N$ lowercase letters where $1 \leq N \leq 2^{12}$. It should have exactly $K$ unique substrings. If there are multiple such strings, any will be accepted. It can be proven that such a string always exists with the given constraints of $N$ and $K$.

## Examples

| standard input | standard output |
| :--- | :--- |
| 15 | banana |
| 351 | abcdefghijklmnopqrstuvwxyz |

## Note

For the first example, the 15 unique substrings of banana are a, an, ana, anan, anana, b, ba, ban, bana, banan, banana, n, na, nan and nana. Another string that has 15 unique substrings is aaaaaaaaaaaaaaa which would also be a correct output for the first example.

## Problem D. Pie Max Flow

| Input file: | standard input |
| :--- | :--- |
| Output file: | standard output |
| Time limit: | 1 second |
| Memory limit: | 256 megabytes |

There is a graph with $N+1$ vertices numbered 0 to $N$. For $1 \leq i \leq N$, an edge with capacity $A_{i}$ connects vertices 0 and $N$. For $1 \leq i \leq N$, an edge with capacity $B_{i}$ connects vertices $i$ and $((i \bmod N)+1)$. There are no other edges in the graph. All edges are undirected.
A flow in a graph from a source vertex $s$ to a sink vertex $t$ assigns to each edge $e=\left(v_{1}, v_{2}\right)$ a direction (either $v_{1} \rightarrow v_{2}$ or $v_{2} \rightarrow v_{1}$ ) and a number $F_{e}$ such that $0 \leq F_{e} \leq K_{e}$, where $K_{e}$ is the capacity of edge $e$. We say that there are $F_{e}$ units of flow in the specified direction, out of $v_{1}$ or $v_{2}$, and into the other vertex. Every flow must satisfy the constraint that for each vertex $v$ except the source and sink, the sum of the flows out of $v$ equals the sum of the flows into $v$. The maximum flow from $s$ to $t$ is the maximum over all flows from $s$ to $t$ of the sum of flows out of $s$ minus the sum of flows into $s$.
Let $C_{i}$ be the maximum flow from vertex 0 to vertex $i$. Find the value of $C_{1}+\cdots+C_{N}$.
Sequences $A$ and $B$ will not be given directly. Instead $N, A_{1}, B_{1}$ and integers $P, Q, W, X$ are given. Then for $i>1$ :

$$
\begin{aligned}
& A_{i}=\left(P A_{i-1}\right) \\
& \bmod Q \\
& B_{i}=\left(W B_{i-1}\right)
\end{aligned} \bmod X
$$

## Input

Line 1 contains one integer $N\left(2 \leq N \leq 2^{20}\right)$.
Line 2 contains three integers $A_{1}, P, Q\left(1 \leq A_{1}, P, Q \leq 2^{29}\right)$.
Line 3 contains three integers $B_{1}, W, X\left(1 \leq B_{1}, W, X \leq 2^{29}\right)$.

## Output

Print one line with one integer, the value of $C_{1}+\cdots+C_{N}$.

## Example

|  | standard input |  | standard output |  |
| :--- | :--- | :--- | :--- | :--- |
| 5 |  | 64 |  |  |
| 7 | 2 | 13 | 3 | 11 |

## Note

For the first example, $A=[7,1,2,4,8], B=[5,4,1,3,9], C=[20,9,7,8,20]$. The graph is shown below.


## Problem E. Odd Colouring

Input file:
Output file:
Time limit:
Memory limit:
standard input
standard output
1 second
256 megabytes

There is a grid with $R$ rows and $C$ columns. Rows are numbered 0 to $R-1$ and columns are numbered 0 to $C-1$. Let the cell at row $r$ and column $c$ be denoted as $(r, c)$. Troy will add $N(N \geq R+C)$ balls to the grid; the $i$-th ball will be added to cell $((i-1) \bmod R,(i-1) \bmod C)$. Ondrej will add $M$ balls to the grid; the $j$-th ball will be added to cell $\left(X_{j}, Y_{j}\right)$. Multiple balls may be added to the same cell.
Every ball will be painted either black or white. Determine the number of ways to paint the balls such that every row and every column has an odd number of black balls. Compute the number of ways modulo 998244353.

## Input

Line 1 contains four integers $R, C, N, M\left(1 \leq R, C, N \leq 2^{29} ; R+C \leq N ; 1 \leq M \leq 2^{16}\right)$.
Line 2 contains $M$ integers $X_{1}, \ldots, X_{M}\left(0 \leq X_{j}<R\right)$.
Line 3 contains $M$ integers $Y_{1}, \ldots, Y_{M}\left(0 \leq Y_{j}<C\right)$.

## Output

Print one line with one integer, the number of ways.

## Examples

| standard input |  | standard output |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 4 | 6 | 2 | 8 |
| 1 | 1 |  |  |  |
| 3 | 0 | 503245988 |  |  |
| 6 | 4 | 50 | 5 |  |
| 0 | 1 | 2 | 3 | 4 |
| 3 | 3 | 3 | 3 | 3 |

## Note

For the first example, Troy added balls to $(0,0),(1,1),(0,2),(1,3),(0,0),(1,1)$ and Ondrej added balls to $(1,3),(1,0)$.

